Introduction Course in Mathematics

(IRC - 1)

FM - 75
PM - 30

GROUP- A (COMPULSORY)

1×5

1. (a) Define Tautology

   (b) Give an example of surjective function which is not injective

   (c) state Fermat's Little theorem

   (d) Give an example of infinite bounded set

   (e) Define limit of a Sequence

2. Find remainder when 3^{100} is divided by 5

3. Test the convergence of the series whose general term is $\sqrt{n^2+1} - n$ 5

Section B

Answer any four questions.

4×15

4. write truth table of $(p \wedge q) \vee \sim r$

(3)

Hint and Solution

1 (a) A statement that is true by necessary or by virtue of logical form

(b) $f = \{(1,2), (2,2), (3,1)\}$; $f : A \to A$; $A = \{1,2,3\}$

(c) $a^p \equiv a \pmod{p}$

(d) $A = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots\}$

(e) $\lim_{n\to\infty} u_n = l$ iff for every given $\varepsilon > 0$ $\exists$ positive integer n_0 such that $|u_n - l| < \varepsilon$ $\forall\ n \geq n_0$

2.
$$3^{100} = (3^4)^{25}$$
$$= (81)^{25}$$
$$\equiv (1)^{25} \pmod{5}$$
$$= 1 \pmod{5}$$

Ans = 1

3.
$$u_n = \sqrt{n^2+1} - n$$

let $v_n = \frac{1}{n}$

then $\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$

$\Rightarrow$ Σu_n and Σv_n have same nature

$\Rightarrow$ Σu_n is ~~convergent~~ divergent.

②

5. Using chinese remainder theorem solve the system of linear congruence

$$x \equiv 3 \pmod{11}$$
$$x \equiv 5 \pmod{19}$$
$$x \equiv 10 \pmod{29}$$

6. Let $A = \{1, 2, 3\}$. List all 1-1 function from A to A

7. Find supremum and infimum of the following set

$\{1 + \frac{1}{2^r}$; r is non-negative integer $\}$

8. Show that ordered field of rational numbers not order - complete

9. Show that the $\{x_n\}$, where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.

10. Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent

Section B

4.

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee \sim r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

5. $m = 11 \cdot 29 \cdot 19 = 6061$

$x_1 = 3, \ x_2 = 5, \ x_3 = 10$

$m_1 = 11, \ m_2 = 19, \ m_3 = 29$

$M_1 = \frac{m}{m_1} = 551$

$M_2 = \frac{m}{m_2} = 319$

$M_3 = \frac{m}{m_3} = 109$

Reduced System is

(5)

$551 x \equiv 1 \pmod{11}$

$319 x \equiv 1 \pmod{19}$

$209 x \equiv 1 \pmod{19}$

$\Rightarrow x_1 = 1 \ , \ x_2 = 14 \ , \ x_3 = 5$

$\Rightarrow \overline{x} = a_1 M_1 x_1 + a_2 M_2 x_2 + a_3 M_3 x_3 \pmod{6061}$

$= 3 \cdot 551 \cdot 1 + 5 \cdot 319 \cdot 14 + 10 \cdot 209 \cdot 5$

$= 4128 \pmod{6061}$

$\Rightarrow \overline{x} = 4128 \pmod{606}$

6. $f_1 = \{ (1,1), (2,2), (3,3) \}$

$f_2 = \{ (1,1), (2,3), (3,2) \}$

$f_3 = \{ (1,3), (2,2), (3,1) \}$

$f_4 = \{ (1,2), (2,1), (3,3) \}$

$f_5 = \{ (1,2), (2,3), (3,1) \}$

$f_6 = \{ (1,3), (2,1), (3,2) \}$

7. $\sup = 2$, $\inf = 1$

8. consider $S = \{x : x \in Q, x \geq 0 \text{ and } x^2 < 2\}$

then it is easy to see that 3 is an upper bound of S

$\Rightarrow$ S is bounded above

Next to show that $3 \notin S$

Next to show that $\sup S = \sqrt{2} \notin S$

This shows the result.

9. $x_1 = 1$

$x_2 = \sqrt{2 + x_1} = \sqrt{3}$

$x_3 = \sqrt{2 + x_2} = \sqrt{2 + \sqrt{3}}$

$\vdots$

$\Rightarrow x_1 < x_2 < x_3 < \cdots$

$\Rightarrow \langle x_n \rangle$ is monotonically increasing function.

$\because$ $x = 1 \leq 2$

let $x_n \leq 2$

then $x_{n+1} = \sqrt{2 + x_n} = \sqrt{2+2} = \sqrt{4} = 2 \leq 2$

$\Rightarrow x_n \leq 2 \quad \forall n \in N$

$\Rightarrow$ 2 is an upper bound ~~of~~ of $\{x_n\}$

$\Rightarrow$ $\{x_n\}$ is converges to its supremum .

let $\lim_{n\to\infty} x_n = l$

Then $l = \sqrt{2+l}$

$\Rightarrow$ $l^2 - l - 2 = 0$

$\Rightarrow$ $l = -1$ or $l = 2$

$\because$ $l \neq -1$

$\therefore$ $l = 2$

10. ~~Show~~ $1 + \frac{1}{2} = 1 + \frac{1}{2}$

$\frac{1}{3} + \frac{1}{4} > \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} > \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 1/2$

$\vdots$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} = \left(1 + \frac{1}{2}\right) + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) + \cdots$$

$$> 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} > 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots \text{ to } \infty$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} \text{ is divergent.}$$