MODEL QUESTION PAPER OF MATHEMATICS SEMESTER-I

INTRODUCTION COURSE IN MATHEMATICS

(IRC – 1)

Full Marks=75

Pass Marks=30

GROUP:-A (COMPULSORY)

1. (a) Define Tautology. $1\times5=5$

(b) Give an example of surjective function which is not injective.

(c) State Fermat's little theorem.

(d) Give an example of infinite bounded set.

(e) Define Limit of a sequence.

2. Find remainder when 3^{100} is divided by 5. (5)

3. Test the convergence of the series whose general term is

$\sqrt{n^2+1}-n$. (5)

GROUP:-B (Answer any four questions) $15\times4=60$

4. Write truth table of $(p\wedge q)\vee(\sim r)$.

5. Using Chinese remainder theorem, solve the system of linear Congruence:

$$x \equiv 3 \pmod{11}$$
$$x \equiv 5 \pmod{19}$$
$$x \equiv 10 \pmod{29}$$

6. Let $A = \{1, 2, 3\}$. List all one-one function from A to A.

7. Find supremum and infimum of the following set:

$\{1 + \frac{1}{2^r}$; r is non-negative integer$\}$.

8. Show that ordered field of rational numbers is not order-complete.

9. Show that the $\{x_n\}$, where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and Converges to 2.

10. Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

HINTS AND SOLUTIONS

1.

(a) A statement that is true by necessary or by virtue of logical term.

(b) $f = \{(1,2),(2,2),(3,1)\}$; $f : A \to A$; $A = \{1,2,3\}$.

(c) $a^p \equiv a \pmod{p}$.

(d) $A=\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}..........\right\}$.

(e) $\lim_{n\to\infty} u_n = l$ iff for every given $\in > 0$,$\exists$ positive integer n_0 such that $|u_n - l| < \in \ \forall n \ge n_0$.

2. $3^{100} = \left(3^4\right)^{25}$

$= (81)^{25}$

$= (1)^{25} \pmod{5}$

$= 1 \pmod{5}$

$= 1$.

3. $u_n = \sqrt{n^2+1} - n$

Let $v_n = \frac{1}{n}$

then $\lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{2}$

$\Rightarrow \sum u_n$ and $\sum v_n$ have same nature.

$\Rightarrow \sum u_n$ is divergent.

GROUP:-B

4.

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee (\sim r)$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

5. $m = 11.29.19 = 6061$

$x_1 = 3, x_2 = 5, x_3 = 10$

$m_1 = 11, m_2 = 19, m_3 = 29$

$$M_1 = \frac{m}{m_1} = 551$$

$$M_2 = \frac{m}{m_2} = 319$$

$$M_3 = \frac{m}{m_3} = 209$$

Reduced system is

$551x \equiv 1 \pmod{11}$

$319x \equiv 1 \pmod{19}$, $209x \equiv 1 \pmod{29}$

$\Rightarrow x_1 = 3, x_2 = 5, x_3 = 10$

$\Rightarrow \bar{x} = a_1 M_1 x_1 + a_2 M_2 x_2 + a_3 M_3 x_3 \pmod{6061}$

$= 3.551.1 + 5.319.14 + 10.209.5$

$= 4128 \pmod{6061}$

$\Rightarrow \bar{x} = 4128 \pmod{6061}$.

6. $f_1 = \{(1,1),(2,2),(3,3)\}$

$f_2 = \{(1,1),(2,3),(3,2)\}$

$f_3 = \{(1,3),(2,2),(3,1)\}$

$f_4 = \{(1,2),(2,1),(3,3)\}$

$f_5 = \{(1,2),(2,3),(3,1)\}$

$f_6 = \{(1,3),(2,1),(3,2)\}$

7. $\sup = 2$, $\inf = 1$.

8. Consider $S = \{x : x \in Q, x \geq 0, x^2 < 2\}$

then it is easy to see that 3 is an upper bound of S.

$\Rightarrow S$ is bounded above.

Next to show that $3 \notin S$.

Next to show that $\sup S = \sqrt{2} \notin S$.

This shows the result.

9. $x_1 = 1$, $x_2 = \sqrt{2 + x_1} = \sqrt{3}$, $x_3 = \sqrt{2 + x_2} = \sqrt{2 + \sqrt{3}}$.............

$\Rightarrow x_1 < x_2 < x_3 <$......................

$\Rightarrow \langle x_n \rangle$ is monotonically increasing function.

$\because x_1 = 1 \le 2$

Let $x_n \le 2$

then $x_{n+1} = \sqrt{2 + x_n} = \sqrt{2 + 2} = \sqrt{4} = 2 \le 2$

$\Rightarrow x_n \le 2 \quad \forall n \in \mathbb{N}$

$\Rightarrow 2$ is an upper bound of $\{x_n\}$.

$\Rightarrow \{x_n\}$ converges to its supremum.

Let $\lim_{n \to \infty} x_n = l$

Then $l = \sqrt{2 + l}$

$\Rightarrow l^2 - l - 2 = 0$

$\Rightarrow l = 2$ or $l = -1$

$\because l \ne -1$

$\therefore l = 2$.

10. $1 + \frac{1}{2} = 1 + \frac{1}{2}$

$\frac{1}{3} + \frac{1}{4} > \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$$\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}>\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}=\frac{1}{2}$$ ………………………………

……………………………………………………………………

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n}=\left(1+\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+\ldots\ldots\ldots\ldots>1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n}>1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\ldots\ldots\ldots\ldots \text{ to } \infty.$$

$$\Rightarrow \sum_{n=1}^{\infty}\frac{1}{n} \text{ is divergent .}$$